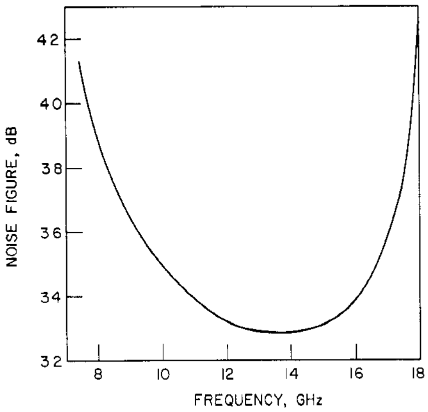

Fig. 4. Noise figure of a Si IMPATT-diode amplifier with a small-signal gain of 20 dB calculated from the noise equivalent circuit.

figure at frequencies above the frequency of maximum negative conductance.

Conclusions

It has been shown that a five lumped-element frequency-independent equivalent circuit can be constructed for IMPATT diodes. It has a driving-point impedance approximately equal to the small-signal impedance of the diode over the frequency range of interest in IMPATT-diode applications. The small-signal noise voltage across the diode can be calculated by incorporating two correlated noise sources in the equivalent circuit which are also frequency-independent. The equivalent circuit elements can be calculated by one of the following methods: 1) numerical determination to fit experimentally measured small-signal impedance (and noise voltage), 2) numerical determination to fit the small-signal impedance (and noise voltage) calculated from an accurate small-signal analysis, and 3) calculation from simple algebraic expressions given here in terms of Read-diode approximation. Frequency dependence of the noise figure of a small-signal IMPATT amplifier has been calculated to illustrate the application of the proposed equivalent circuit.

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Finite-Boundary Corrections to the Coplanar Waveguide Analysis

M. E. DAVIS, E. W. WILLIAMS, AND A. C. CELESTINI

***Abstract*—Conformal mapping calculations of impedance and effective dielectric constant are presented for coplanar waveguide (CPW) lines with finite-substrate thickness. These calculations and experimental data show a departure from the infinite dielectric approximation as the substrate thickness approaches the guide slot width. The quasi-TEM approximation is retained and calculations of static energy density within the substrate are given. This approximation agrees well with field calculations using a finite-element solution to Laplace's equation.**

Introduction

Calculations of wave impedance and effective dielectric constant of the coplanar waveguide (CPW) structure have assumed that the substrate was infinite in extent [1]. This allowed a simple conformal mapping transformation of the field patterns, using complete elliptic integrals, into a homogeneous rectangular configuration. A few investigations bore out the assumption that a finite substrate would not affect the wave propagation for simple waveguide structures [2], [3], but no extended circuitry has been reported that gives quantitative support of these assumptions. In fact, recent investigations into the CPW show marked deviation from the idealized model under some common experimental conditions [4].

As a result, a detailed calculation of the wave impedance has been carried out, still within the zeroth-order approximation of a quasi-TEM structure. By using the familiar Schwarz–Christoffel transformation of the waveguide shown in Fig. 1, the lower half of the Z plane is mapped into the rectangle in the W plane. The transformation characterizing this mapping is [1]

$$\frac{dw}{dz} = \frac{A}{(z^2 - a_1^2)^{1/2}(z^2 - b_1^2)^{1/2}} \tag{1}$$

where A is a constant to be evaluated. The boundaries of the $y=0$ line can be determined in the W plane upon integration

$$W = a + jb = \int_0^{b_1} \frac{A\, dz}{(z^2 - a_1^2)^{1/2}(z^2 - b_1^2)^{1/2}}. \tag{2}$$

The above equation is one form of an elliptic integral. The ratio a/b can be conveniently expressed in terms of tabulated complete elliptic integrals.

$$\frac{a}{b} = \frac{K(k)}{K'(k)} \tag{3}$$

where $k = a_1/b_1$, $K'(k) = K(k')$, and $k' = (1-k^2)^{1/2}$.

This is the point at which most analyses stop. By assuming a semi-infinite dielectric, in parallel with a half-space of air, the equivalent static capacitance per unit length for a pure TEM mode propagating in the line is

$$C = (\epsilon_r + 1)\epsilon_0 \frac{2a}{b} = 2(\epsilon_r + 1)\epsilon_0 \frac{K(k)}{K'(k)}. \tag{4}$$

This analysis can be extended to the case of finite-substrate thickness by mapping the substrate bottom into the W plane. This will appear approximately as an ellipse in the rectangle, as shown in Fig. 1. The mapping of complex arguments in elliptic functions is reasonably straightforward [5]. Equation (2) can be rewritten in the descriptive form

$$W = F(\phi, k) = \int_0^{\phi} \frac{d\theta}{(1 + k^2 \sin^2 \theta)} \tag{5}$$

where $F(\phi, k)$ is an incomplete elliptic integral of the first kind with amplitude ϕ and modulus k. In our case $z = \sin \phi$. Since we can choose the constant A arbitrarily without affecting generality, it is given a

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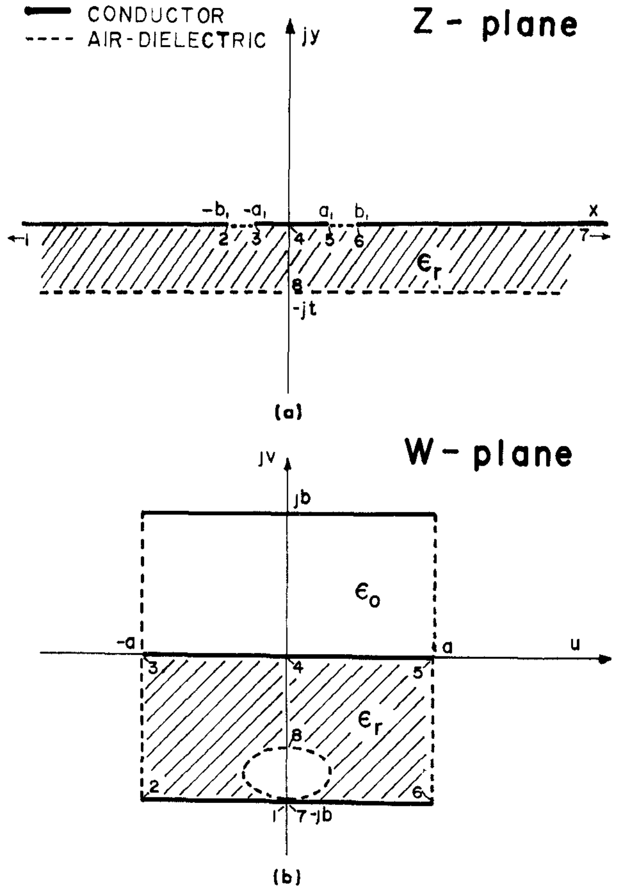


Fig. 1. Conformal mapping of CPW with finite substrate into W plane.

particular value to normalize values of z to a_1. When we evaluate the axis $y=jt$ in the transformation, z is complex and we separate $F(\phi, k)$ into real and imaginary parts:

$$W = F(\phi + j\theta, m) = F(\alpha, m) + jF(\beta, m_1) \tag{6}$$

where $z=\sin(\phi+j\theta)$, $m=k^2$, $m_1=(1-m)$, and α and β are amplitudes to be evaluated. From [5] we see $\cot^2\alpha$ is the positive root of the equation

$$x^2 - [\cot^2\phi + m\sinh^2\theta\csc^2\phi - m_1]x - m_1\cot^2\phi = 0 \tag{7a}$$

and β is found from

$$m\tan^2\beta = \tan^2\phi\cot^2\alpha - 1. \tag{7b}$$

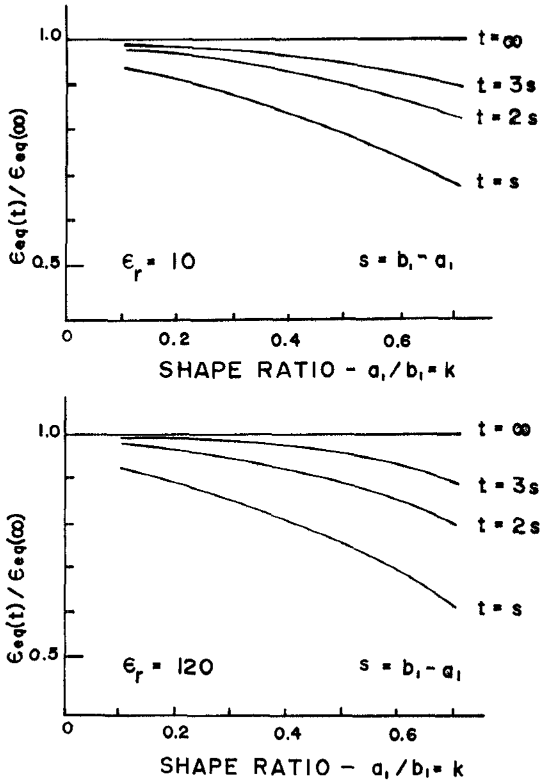
Now, since for our mapping

$$z = x + jy = \sin(\phi + j\theta) \tag{8}$$

we obtain, after some trigonometric manipulation, relations between (θ, ϕ) and the known quantities (x, y)

$$\sin\phi = x/\cosh\theta$$
$$\cos\phi = y/\sinh\theta.$$

On substituting into (7a) and (7b) we obtain the necessary values of α and β. Numerical values for the incomplete elliptic integrals were computed using the method of arithmetic–geometric mean [5]. Equation (6) was then solved for the air-dielectric boundary at various values of shape ratio $k=a_1/b_1$, and for thicknesses of one, two, and three times the slot width (b_1-a_1). The equivalent dielectric constant was then numerically calculated using this "void" in the mapping as a distributed shunt–series capacitance in the conformal mapping [6]. Fig. 2 shows the effective dielectric constant calculated in this way for $\epsilon_r=10$ and $\epsilon_r=120$. Using this, the characteristic impedance of the lines was calculated as a function of substrate thickness for several dielectric constants. These results are shown in Fig. 3. Good quantitative agreement with Wen's infinite-substrate calculations [1] is shown for thicknesses greater than twice the slot width (b_1-a_1).



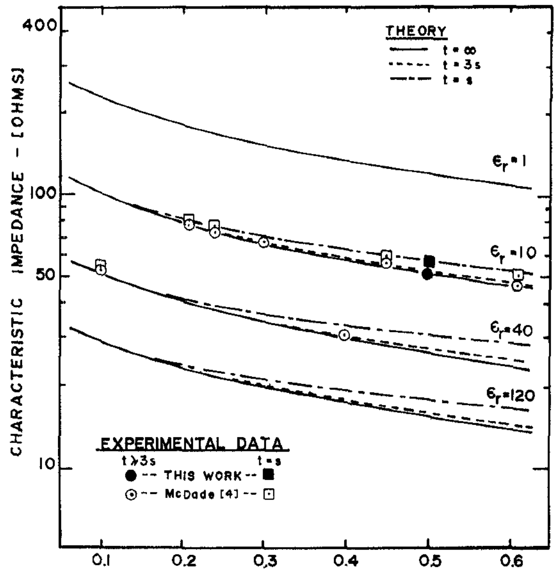


Fig. 3. Characteristic impedance corrections due to finite-substrate thickness.

A measure of the ability of the CPW to guide the microwave signal within the desired circuits is the percentage of energy inside the dielectric. For a dielectric constant $\epsilon_r=10$ the static approximation predicts 91 percent of the energy will be contained in the dielectric. Table I compares the energy density, characteristic impedance, and relative phase velocity predicted by the conformal mapping calculations, to a direct calculation using a finite-element solution to the Laplace equation [7]. As is apparent from both calculations, there is a rapid increase in amount of energy contained

TABLE I

CALCULATION OF CPW TRANSMISSION LINE[a]

	Infinite Substrate Approximation	Thickness = (b_1-a_1)		Thickness = $3(b_1-a_1)$	
		Finite Substrate Conformal Mapping	Finite Element Field Map	Finite Substrate Conformal Mapping	Finite Element Field Map
Relative Phase Vel. v_{ph}	0.43	0.48	0.50	0.43	0.44
Characteristic Impedance, Z_0	51.4	57.7	58.5	52.4	53.0
Percent of Energy in Substrate	91.1	79.0	80.0	89.0	90.0

[a] Characteristics for $\epsilon_r = 10$. $(a_1/b_1 = 0.5.)$

below the dielectric boundary as the slot-width to substrate-thickness ratio decreases.

MEASUREMENTS

A number of CPW lines were fabricated on alumina substrates. The line impedances were measured on an Hp 1815B time-domain reflectometer with a 28-ps rise-time pulse. These results are the solid data points shown on Fig. 3 for the lines designed for 50 Ω from Wen's evaluation $(a_1/b_1 = 0.5)$. The other data points were obtained by McDade [4]. As can be seen, good qualitative results were obtained. It was quite necessary, however, in making the measurements to assure that the substrates were suitably suspended in air, since any metal ground planes affected the impedance measurements. These results were substantiated by McDade [4] who fabricated similar circuits both with and without ground planes. Those circuits with ground planes showed a marked preference for microstrip modes and impedance levels as opposed to CPW.

CONCLUSIONS

The effects of finite-substrate thickness on the characteristics of CPW are important in designing circuits where close control of the transmission-line impedance is needed. It is shown that deviation from the results of an infinite dielectric can approach 10–15 percent for substrates whose thicknesses are less than two times the gap width. These calculations were all made using a static TEM approximation and further corrections would be present if dispersion were considered. Finally, the circuits fabricated showed a pronounced tendency to be affected by external metallic walls, which would be a highly undesirable effect if use of these transmission lines as microwave integrated circuits is anticipated.

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Letters

Analysis of Microwave Circuit for Characterization of Negative-Conductance Devices by Transients

K. TOMIZAWA, T. HARIU, AND H. HARTNAGEL

Abstract—**The assumptions required for the transient method of measuring Gunn-diode conductances are shown to be valid if either the diode susceptance or the characteristic admittance of the resonator transmission line are larger than the modulus of the negative conductance of the device.**

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A method for measuring the negative conductance of Gunn diodes from the envelope of the transient amplitude of oscillations was reported recently [1], [2]. This new method is useful since the measurement of the diode conductance has been possible so far, in those cases where either devices can be stabilized by heavy loading or where a reasonably correct load impedance of oscillating diodes can be measured separately, when only the steady-state conductance is obtained.

The conductance measurement by transient recording depends on the assumptions that the ratio of the voltage across the diode